Modeling Conscious–Unconscious Neural Dynamics with Operator Theory

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Introduction

How much of neural dynamics is intrinsic, and how much arises from unresolved context, feedback, and noise?

Theory Intuition

Closed Koopman models describe coherent, Markovian structure.

General Koopman models describe coherent, though noisy Markovian structure.

Mori–Zwanzig operators explicitly encode memory and stochastic forcing induced by unresolved variables.

Hierarchical Goal

This hierarchy enables interpretable separation of baseline neural dynamics from contextual modulation, offering a principled lens on state-dependent neural computation.

Prior Approaches

Data-driven modeling of neural dynamics has been dominated by Koopman operator theory and related methods such as dynamic mode decomposition (DMD) and HAVOK. These approaches lift nonlinear neural activity into a linear operator framework.

While effective for capturing dominant dynamics, they treat unresolved influences as external forcing or residual noise.

Data Driven Approach

The evolution of ^ is

$$\hat{\mathbf{g}}(t_{n+1}) = \sum_{k=0}^{\ell} \mathcal{L}^{(k)} \cdot \hat{\mathbf{g}}(t_n) + \mathbf{F},$$

where $\mathcal{L}^{(k)}$ incorporates the Markov and dissipative memory terms and incorporates the fluctuating force.

An operator theory hierarchy to model neural dynamics.

Koopman Closure

$$\frac{d}{dt} g(t) = \mathcal{K} g(t). \tag{1}$$

Koopman Operator Theory

$$\frac{d}{dt} g(t) = \mathcal{K} g(t) + \eta(t). \tag{2}$$

Mori-Zwanzig Operator Theory

$$\frac{\partial}{\partial t}\hat{g}(t) = \underbrace{P\mathcal{L}\hat{g}(t)}_{\text{Markov}} + \underbrace{\int_0^t P\mathcal{L}e^{(t-s)Q\mathcal{L}} Q\mathcal{L}\hat{g}(s)\, ds}_{\text{Dissipative Memory}} + \underbrace{P\mathcal{L}e^{tQ\mathcal{L}} Q g(0)}_{\text{Fluctuating Force}}. \tag{3}$$

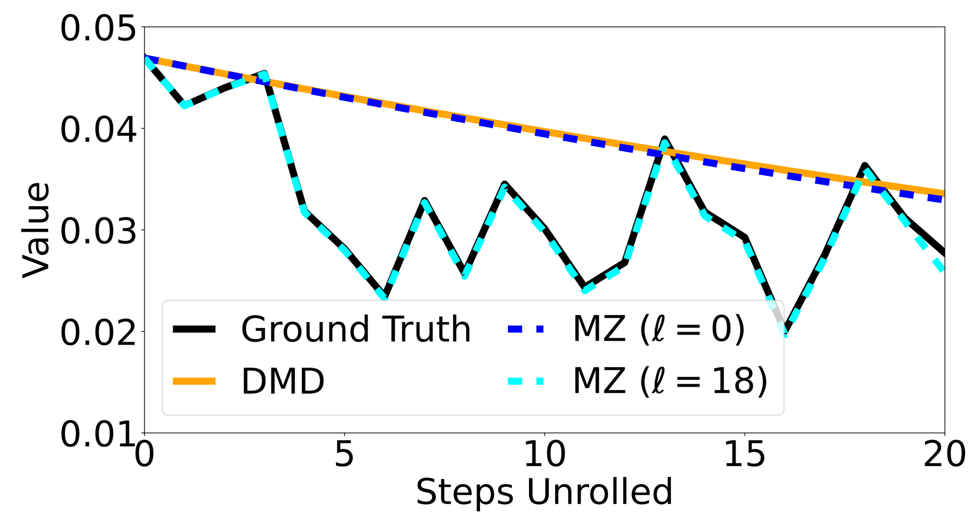
Ornstein-Uhlenbeck

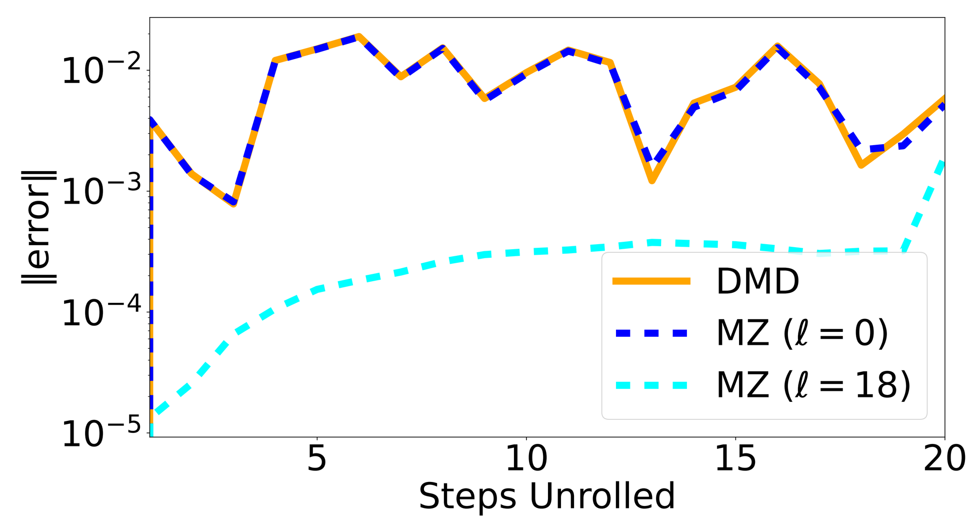
In anesthesia modeling, OU processes serve as interpretable drivers of the dynamics of the neural population.

Mathematically, the OU process is a continuous-time mean-reverting SDE

$$\frac{dx_t}{dt} = -\theta x_t + \sigma \eta(t)$$

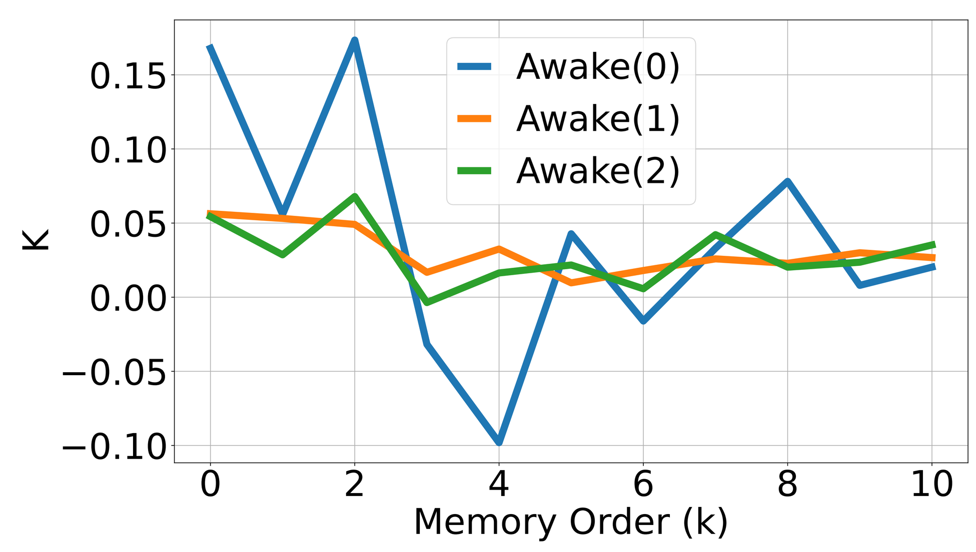
where $\eta(t)$ is the derivative of the Wiener process dW_t/dt.

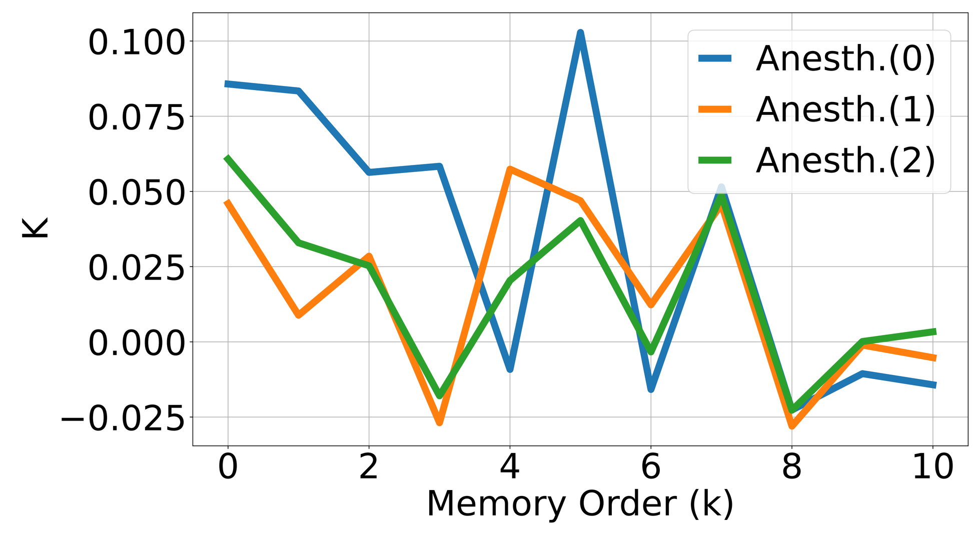




Propofol induced LOC

Model	Awake (MSE)	Anes. (MSE)
DMD	3.140	0.620
MZ	0.101	0.011





Key Take-aways

1. Koopman/DMD extracts coherent, approximately Markovian structure, but misses structured effects of unresolved variables.
2. Mori–Zwanzig adds memory + stochastic forcing, yielding improved rollouts and a more interpretable decomposition of "residual" dynamics.
3. In propofol LOC, learned memory structure differs between awake and anesthetized states, consistent with disrupted feedback and coordination.

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